The Paretian Optimum

Pareto-Optimal Composition of Outputs and Perfect Competition:

It gives us that the point where the condition for equilibrium commodity combination is satisfied is the point of tangency between the PPC curve and line of slope $-p_1/p_2$. In Fig. 21.5, AB is this line, say, and it has touched the PPC curve at the point E. Therefore, the society's equilibrium production point is the point E, and it should produce q^0_1 and q^0_2 of the two commodities.

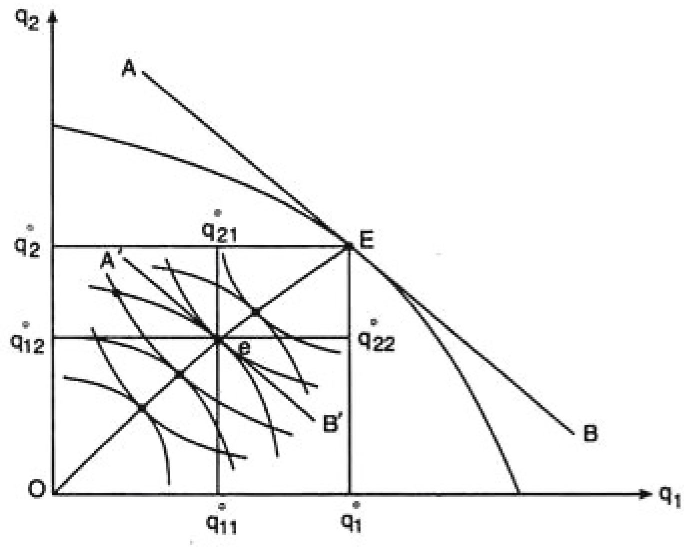

Fig. 21.5 Production sector's plans are consistent with the household sector's plans

We may now come to the distribution of the goods between the two consumers, I and II. They have to be so distributed that the Pareto-efficiency in consumption is achieved, i.e., the marginal condition for such efficiency is satisfied.

As we know, this marginal condition is:

$$MRS^{I}_{Q_1,Q_2} = MRS^{II}_{Q_1,Q_2} \qquad (21.11)$$

We also know that the satisfaction of this condition is guaranteed under perfect competition, since both of them would be equal to p_1/p_2 which is given and constant:

$$MRS^{I}_{Q_1,Q_2} = MRS^{II}_{Q_1,Q_2} = \frac{p_1}{p_2} \qquad (21.17)$$

In Fig. 21.5, the Pareto-efficient distribution of the goods is obtained at the point e on the Edgeworth contract curve for consumption (CCC), for, at this point, both the indifference curves (ICs) of the two consumers have touched the line A'B' which is parallel to the line AB.

That is, in order to obtain the Pareto-efficient distribution of the goods, we have to find out the point (like e) on the Edgeworth CCC at which the numerical slopes of the ICs of the two consumers are equal to p_1/p_2 which is here the numerical slope of the line AB.

To be more specific, as the solution of eqn. (21.26), we have obtained the economy's production of the two goods to be $E(q^0{}_1 q^0{}_2)$ and by solving (21.17), we would obtain the distribution of these quantities between the two consumers (at the point e) to be $(q^0{}_{11}, q^0{}_{12})$ for the first consumer and $(q^0{}_{21}, q^0{}_{22})$ for the second consumer.

Mathematical proof of $MRPT_{Q_2 \text{ into } Q_1} = \frac{MC_{Q_1}}{MC_{Q_2}}$

By definition, at any point on the PPC, we have

$$MRPT_{Q_2 \text{ into } Q_1} = -\left.\frac{dq_2}{dq_1}\right|_{PPC} \quad (1)$$

Now, by definition

$$MC_{Q_1} = \frac{d(TC_{Q_1})}{dq_1} \text{ and}$$
$$MC_{Q_2} = \frac{d(TC_{Q_2})}{dq_2} \quad (2)$$

From (2), we obtain

$$\frac{MC_{Q_1}}{MC_{Q_2}} = \frac{d(TC_{Q_1})}{d(TC_{Q_2})}\frac{dq_2}{dq_1} \quad (3)$$

But $d(TC_{Q_1}) = r_1 dx_{11} + r_2 dx_{12}$ and

$d(TC_{Q_2}) = r_1 dx_{21} + r_2 dx_{22}$ (4)

Hence, $$\frac{d(TC_{Q_1})}{d(TC_{Q_2})} = \frac{r_1 dx_{11} + r_2 dx_{12}}{r_1 dx_{21} + r_2 dx_{22}} \quad (5)$$

Now, for a movement along the PPC, the factors released from the decrease in q_2 must be equal to the factors absorbed by the increase in q_1, i.e.,

$$dx_{11} = -dx_{21} \text{ and}$$
$$dx_{12} = -dx_{22} \quad (6)$$

Substituting from (6) into (5), we have

$$\frac{d(TC_{Q_1})}{d(TC_{Q_2})} = \frac{r_1(-dx_{21}) + r_2(-dx_{22})}{r_1 dx_{21} + r_2 dx_{22}} = -1 \quad (7)$$

Finally, substituting (7) into (3), we obtain

$$\frac{MC_{Q_1}}{MC_{Q_2}} = (-1)\frac{dq_2}{dq_1} = -\frac{dq_2}{dq_1}$$

$$= \text{numerical slope of PPC} = MRPT_{Q_2 \text{ into } Q_1}$$

i.e., $MRPT_{Q_2 \text{ into } Q_1} = \frac{MC_{Q_1}}{MC_{Q_2}}$